Period 1, Oct 28, 2024

b) $y = 2xe^x - e^{2x}$

$$\frac{dy}{dx} = 2 \cdot e^x + 2xe^x - 2e^{2x}$$

$$y = e^{2x} \Rightarrow y = e^u$$

$u = 2x$ $\qquad \frac{dy}{du} = e^u$

$\frac{du}{dx} = 2$

$$\frac{d\cancel{u}}{dx} \cdot \frac{dy}{d\cancel{u}} = 2 \cdot e^u = \underbrace{2 \cdot e^{2x}}$$

b) $y = \sqrt[3]{\frac{(x-2)^3(x^2+5)}{(2x+3)^3}}$

$$\ln y = \ln\left[\frac{(x-2)^3(x^2+5)}{(2x+3)^3}\right]^{\frac{1}{3}} = \frac{1}{3}\ln\left[\frac{(x-2)^3(x^2+5)}{(2x+3)^3}\right]$$

$$\ln y = \frac{1}{3}\left[\ln(x-2)^3 + \ln(x^2+5) - \ln(2x+3)^3\right]$$

$$\frac{1}{3}\left[3\ln(x-2) + \ln(x^2+5) - 3\ln(2x+3)\right]$$

$$\ln y = \ln(x-2) + \frac{1}{3}\ln(x^2+5) - \ln(2x+3)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x-2} \cdot 1 + \frac{1}{3} \cdot \frac{1}{x^2+5} \cdot 2x - \frac{1}{2x+3} \cdot 2$$

$$\cancel{y} \cdot \frac{1}{\cancel{y}}\frac{dy}{dx} = \left[\frac{1}{x-2} + \frac{2x}{3(x^2+5)} - \frac{2}{2x+3}\right] \cdot y$$

$$\frac{dy}{dx} = \left[\frac{1}{x-2} + \frac{2x}{3(x^2+5)} - \frac{2}{2x+3}\right]\left[\sqrt[3]{\frac{(x-2)^3(x^2+5)}{(2x+3)^3}}\right]$$

a) $y = (\sin x)^x$

$\ln y = \ln (\sin x)^x = x \ln(\sin x)$

$\ln y = x \ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln \sin x + x \cdot \frac{1}{\sin x} \cdot \cos x = \ln \sin x + \frac{x \cos x}{\sin x}$

$\cancel{y} \cdot \frac{1}{\cancel{y}}\frac{dy}{dx} = [\ln(\sin x) + x \cot x] \cdot y$

$\frac{dy}{dx} = [\ln(\sin x) + x \cot x][(\sin x)^x]$

$g(x) = F(x)\,h(x) \Rightarrow F(x) = 2^x,\ h(x) = \log_3 \sqrt{x-1}$

3.Find $g'(x)$ if $g(x) = 2^x \cdot \log_3 \sqrt{x-1}$.

$g(x) = 2^x \cdot \log_3 (x-1)^{\frac{1}{2}}$

$y = 2^x$

$\ln y = \ln 2^x = x \ln 2$

$\ln y = x \cdot \ln 2$

$\cancel{y} \cdot \frac{1}{\cancel{y}}\frac{dy}{dx} = \ln 2 \cdot y$

$\frac{dy}{dx} = (\ln 2)(2^x)$

$g'(x) = (\ln 2)\, 2^x \log_3 (x-1)^{\frac{1}{2}}$

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$2^x \left(\frac{1}{2(x-1)\ln 3}\right)$

$y = \log_3 (x-1)^{\frac{1}{2}} \Rightarrow 3^y = (x-1)^{\frac{1}{2}}$

$\ln 3^y = \ln (x-1)^{\frac{1}{2}} \Rightarrow y \cdot \ln 3 = \frac{1}{2} \ln(x-1)$

$\frac{1}{\ln 3} \cdot \cancel{\ln 3} \cdot \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{(x-1)} \cdot \frac{1}{\ln 3} \Rightarrow \frac{dy}{dx} = \frac{1}{2(x-1)\ln 3}$

4. $\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u}\frac{du}{dx}$

0. Find $\frac{d^2y}{dx^2}$ if $y^3 + y = 2\cos x$

$$3y^2\frac{dy}{dx} + \frac{dy}{dx} = 2(-\sin x)$$

$$\frac{\frac{dy}{dx}\cancel{(3y^2+1)}}{\cancel{(3y^2+1)}} = \frac{-2\sin x}{(3y^2+1)}$$

$$\frac{dy}{dx} = \boxed{\frac{-2\sin x}{(3y^2+1)}}$$

$$\frac{d^2y}{dx^2} = \frac{(-2\cos x)(3y^2+1) - (-2\sin x)(6y\frac{dy}{dx})}{(3y^2+1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(-2\cos x)(3y^2+1) + (2\sin x)(6y(\frac{-2\sin x}{3y^2+1}))}{(3y^2+1)^2}$$

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$$g(x) = F(x)\cdot h(x)$$

$$g'(x) = F'(x)\cdot h(x) + F(x)\cdot h'(x)$$

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## Example 3

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\cdot\frac{(\sqrt{1+x}+1)}{(\sqrt{1+x}+1)}$$

$$\frac{\sqrt{1+0}-1}{0}=\frac{\sqrt{1}-1}{0}=\frac{1-1}{0}=\frac{0}{0}=\emptyset$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}[\sqrt{1+x}-1]}{\frac{d}{dx}[x]}=\frac{\frac{d}{dx}[(1+x)^{\frac{1}{2}}-1]}{\frac{d}{dx}[x]}$$

$$\lim_{x\to 0}\frac{\frac{1}{2(\sqrt{1+x})}\cdot 1-0}{1}=\frac{\frac{1}{2\sqrt{1}}\cdot 1-0}{1}=\frac{1}{2}$$

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$$\lim_{x\to 0}\frac{e^{2x}-1}{x}=\frac{e^{2\cdot 0}-1}{0}=\frac{1-1}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{2e^{2x}-0}{1}=\frac{2\cdot e^{2\cdot 0}}{1}=\frac{2\cdot e^{0}}{1}=\frac{2\cdot 1}{1}=2$$

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$$\lim_{x\to\infty}\frac{\ln x}{x} \Rightarrow \frac{\ln\infty}{\infty} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\ln x}{x} = \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = \frac{\frac{1}{\infty}}{1} = \frac{0}{1} = 0$$

$$\lim_{x\to-\infty}\frac{x^2}{e^{-x}} = \frac{(-\infty)^2}{e^{-(-\infty)}} = \frac{\infty^2}{e^{\infty}} = \frac{\infty}{\infty}$$

$$\lim_{x\to-\infty}\frac{x^2}{e^{-x}} = \lim_{x\to-\infty}\frac{2x}{-e^{-x}} = \frac{2(-\infty)}{-e^{-(-\infty)}} = \frac{-2\cdot\infty}{-e^{\infty}} = \frac{-\infty}{-\infty}$$

$$\lim_{x\to-\infty}\frac{x^2}{e^{-x}} = \lim_{x\to-\infty}\frac{2x}{-e^{-x}} = \lim_{x\to-\infty}\frac{2}{e^{-x}} = \frac{2}{e^{-(-\infty)}} = \frac{2}{e^{\infty}} = \frac{2}{\infty} = 0$$

$$\lim_{x\to\infty}\left(x\sin\frac{1}{x}\right) = \infty \cdot \sin\frac{1}{\infty} = \infty \cdot \sin 0 = \infty \cdot 0$$

$$\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \frac{\sin 0}{0} = \frac{0}{0} = \varnothing$$

$$\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{d}{dx}\left[\sin\frac{1}{x}\right]}{\frac{d}{dx}\left[\frac{1}{x}\right]} = \lim_{x\to\infty} \frac{\left(\cos\frac{1}{x}\right)\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}}$$

$$\frac{1}{\infty} = 0$$

$$\cos 0 = 1$$

$$y = \sin\frac{1}{x} = \sin x^{-1} \Rightarrow y = \sin u$$

$$u = x^{-1} \qquad \frac{dy}{du} = \cos u$$

$$\frac{du}{dx} = -1x^{-1-1} = -1x^{-2} = \frac{-1}{x^2}$$

$$\frac{du}{dx} \cdot \frac{dy}{du} = \frac{dy}{dx} = \left(\frac{-1}{x^2}\right)\cos\left(\frac{1}{x}\right)$$

$$1 = \lim_{x\to\infty} x^{1/x} = \infty^{\frac{1}{\infty}} = \infty^{RSN}$$

$$y = \lim_{x\to\infty} x^{\frac{1}{x}}$$

$$\ln y = \ln\left[\lim_{x\to\infty} x^{\frac{1}{x}}\right] = \lim_{x\to\infty} \ln\left(x^{\frac{1}{x}}\right) = \lim_{x\to\infty} \frac{1}{x}\cdot \ln x$$

$$0$$

$$= \lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1}$$

$$\ln_e y = 0 \Rightarrow e^0 = y$$

$$\frac{\frac{1}{\infty}}{1} = \frac{0}{1} = 0$$

$$1 = y$$

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$$\lim_{x\to 0^+} x^x = 0^0 = ? = RSN^{RSN}$$

$$y = \lim_{x\to 0^+} x^x \Rightarrow \ln y = \lim_{x\to 0^+} \ln x^x = \lim_{x\to 0^+} x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \frac{\cancel{\frac{1}{x}}}{\left(\frac{-1}{x}\right)\cancel{\left(\frac{1}{x}\right)}}$$

$$\ln y = 0$$

$$e^0 = y = 1$$

$$= \lim_{x\to 0^+} \frac{1}{\frac{-1}{x}} = \frac{1}{\frac{-1}{RSN}}$$

$$\frac{1}{-\infty} = 0$$

$$\lim_{x\to c} f(x) = -14 \qquad \lim_{x\to c} g(x) = -10$$

$$\lim_{x\to c}\left[f^2(x) - \tfrac{1}{2}g(x)\right] = (-14)^2 - \tfrac{1}{2}(-10) = 196 + 5 = 201$$

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$$5(-14) - (10)^2 = -70 - 100 = -170$$

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$$\lim_{x\to 0^-} x^8 - \frac{1}{x} = 0^8 - \frac{1}{0^-} = 0 - \frac{1}{-RSN} = +\frac{1}{RSN} = \infty$$

$$\emptyset$$

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$$\lim_{x\to 0^-} x^5 - \frac{1}{x} = (-RSN)^5 - \frac{1}{-RSN} = 0 + \frac{1}{RSN} = \infty$$

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$$\lim_{x\to 0}\frac{1-\cos 2x}{3x} = \lim_{x\to 0}\frac{1-\cos 2x}{\frac{3}{2}(2x)} = \frac{2}{3}\cdot 0 = 0$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

$$\lim_{x\to 0}\frac{1-(1-2\sin^2 x)}{2x} =$$

$$\lim_{a\to 0}\frac{\sin a}{a} = 1$$

$$\lim_{x\to 0}\frac{\cancel{2}\sin^2 x}{\cancel{2}x} = \boxed{\frac{\sin x}{x}}\cdot\frac{\sin x}{1}$$

$$1\cdot 0 = 0$$

$$\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \lim_{x\to 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{x} = \lim_{x\to 0} \frac{\sin^2 x}{x} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1} = 1 \cdot 0 = 0$$

$$\lim_{\Delta x\to 0} \frac{\left(\sqrt{x+\Delta x} - \sqrt{x}\right)}{\Delta x} \cdot \frac{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} =$$

$$\lim_{\Delta x\to 0} \frac{\cancel{x}+\Delta x + \cancel{\sqrt{x}\cdot\sqrt{x+\Delta x}} - \cancel{\sqrt{x}\cdot\sqrt{x+\Delta x}} - \cancel{x}}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} = \lim_{\Delta x\to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\lim_{x\to 0} \frac{\sin 5x}{\sin 3x} = \lim_{x\to 0} \frac{15x \sin 5x}{15x \sin 3x} = \lim_{x\to 0} \frac{5 \cdot \boxed{3x \cdot} \boxed{\sin 5x}}{3 \cdot \boxed{\sin 3x} \boxed{\cdot 5x}}$$

$$\frac{5}{3} \qquad 1 \qquad 1$$

$$\lim_{x\to0}\frac{x}{\sin x}=1=\lim_{x\to0}\frac{\sin x}{x}$$

$$\frac{5}{5}=1$$

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$$\frac{12x\,\sin 4x}{12x\,\sin 3x}=\frac{4}{3}$$

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$$\lim_{x\to0}\frac{3x^2\,(1+\cos 5x)}{(1-\cos 5x)(1+\cos 5x)}=\lim_{x\to0}\frac{3x^2(1+\cos 5x)}{1-\cos^2 5x}$$

$$\lim_{x\to0}\frac{25\cdot 3x^2(1+\cos 5x)}{25\cdot\sin^2 5x}=\lim_{x\to0}\frac{3\cdot 5x\cdot 5x\cdot(1+\cos 5x)}{25\cdot\sin 5x\cdot\sin 5x}$$

$\cos 0=1$

1 1

$$\frac{3(1+1)}{25\cdot1\cdot1}=\frac{6}{25}$$

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